

CREATIVE INVESTIGATIONS **IN EARLY MATH**

Angela Eckhoff, PhD



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Introduction

“Look at me!” shouts Clay to his friends Shawn and Cassidy. The two children are standing down below as Clay climbs a small set of stairs leading to a higher platform at the Nashville Zoo’s Jungle Gym feature. The Jungle Gym includes a 35-foot-tall wooden Tree of Life climbing structure designed to encourage children to explore different zoo animals’ methods of locomotion. “You climb like a monkey!” observes Clay’s teacher, Kathleen, who is standing nearby. “C’mon, climb like monkeys,” encourages Clay to his two friends waiting at the bottom of the climbing structure. The children quickly climb up to the platform where Clay is waiting. Together, the trio decides to climb to the rope netting to swing like monkeys. As the children navigate their way across the rope netting with monkey-inspired movements, Kathleen narrates their actions using position words: “Awesome, Clay! You are nearing the top.” “Shawn is beside Cassidy now.” “Keep going to the right. You’ve got it!”

From his vantage point on the rope netting, Shawn spots a large, open-mouthed snake structure on the other side of the Jungle Gym. After Shawn points out his discovery to his friends, he encourages the others to join him inside the snake. The three begin to make their way off the rope netting to the wooden platform. Watching the trio from the ground, Kathleen encourages Shawn to think about the movement of a snake. “You’re going to have to change from a monkey to a snake. What was the word the zookeeper taught us about how snakes move?” She prompts, “It starts with an s.” “Slither! Slowly slither,” recalls Shawn. Kathleen moves with the group over to the snake and encourages them to enact slithering movements as she uses more position words. “Slither through there. Cassidy is slithering on top of the snake! Yikes, these snakes keep slithering down to the ground!”

As the experiences of Clay, Shawn, and Cassidy demonstrate, everyday activities can provide playful opportunities for teachers to introduce and reinforce math skills. In this example, Kathleen was able to narrate the children’s natural movements using descriptive position words that are central to understanding spatial relationships. She supported the children’s creative play and worked to encourage them to apply what they knew about



Children find interesting ways to climb up the Nashville Zoo's Jungle Gym.



The snake head at the Nashville Zoo inspires children to move around in a slithering motion.

animal locomotion while providing the language of geometry to extend and support their understandings.

Preschoolers can learn math concepts naturally as they become intrigued with learning about their world and how it works. As an educator, you can fuel young children's desire for knowledge and intentionally encourage creative explorations that help preschoolers build math skills every day in the different environments they encounter. This book is designed to provide you with knowledge and lesson ideas that support young children's development of mathematics understanding via inquiry and creative-thinking skills.

Creative-thinking skills can be found in all aspects of young children's playful engagement and include imagining, exploration, visualization, design, divergent thinking, and solution finding. Teaching mathematics in a linear, fact-based fashion is not ideal for children ages three to five. Instead, they will thrive in classrooms that promote hands-on experimentation and intentionally encourage minds-on learning experiences. Young children build early mathematics understandings through guided and independent investigations that encourage them to be actively involved in meaningful ways. These investigations can be large and sustained over a period of time—such as creating maps of the classroom or school—or they can be part of a planned lesson activity—such as helping measure ingredients to make playdough. The most important aspect of these investigations is that the children are engaged both physically and mentally—hands-on and minds-on engagement. This book highlights the role of the early childhood educator as guide

and facilitator in the planning, implementation, and assessment of creative mathematics experiences. *Creative Investigations in Early Math* will support your development of these types of classroom experiences in the following ways:

- You can link mathematics content and inquiry-based learning.
- You can plan cooperative math lessons that will engage all children in your classroom.
- You can find ways to support children's math learning in everyday classroom experiences.
- You can recognize children's mathematical thinking in order to build on their current levels of understanding.
- You can document children's knowledge development with a variety of classroom work samples.

Assessment is an important element in mathematics teaching and learning, as you can gain insight into what works with particular children and what new strategies you can try as you strive to promote more-advanced understanding. Conversations, children's drawings, and other types of classroom work are all representative forms of children's thinking and understanding. By exploring children's work to better understand their thinking, you can create experiences that build upon and deepen children's mathematical understanding.

Playful Learning

Play is an important element for learning in early childhood because it helps young children learn about themselves, their environment, people they interact with, and the world around them. Playful learning encourages children to explore and experiment in situations in which they feel comfortable taking risks and delving into the unknown. When children explore, experiment, and cooperate through play, they learn about how the world works. Children need teachers who are supportive of children's play and who work to carefully identify play situations where teacher guidance or involvement will be welcome and positive.

Young children make use of the information gathered during their everyday experiences by bringing these ideas into their play to further experiment and clarify their understanding. This process is child driven, and your role is to support preschoolers' curiosity. Using intentional pedagogical practices, you can create early childhood classrooms that honor the ways in which children learn, explore, and play. As you carefully observe children's play and recognize their current stages of understanding, you can scaffold their thinking by questioning, supplying materials that encourage experimentation, and providing opportunities for guided learning.

Early Mathematics Experiences

Young children love to actively explore as they seek to understand what is going on around them. As they investigate their environment, they can build mathematical knowledge during everyday interactions. For example, when preschoolers place one plate in front of each child at lunch, sing counting songs, or play board games with friends, they are building their content knowledge as well as their mathematical identities. The concept of *positive mathematics identities* is important because young children's early experiences with mathematics can influence their confidence in their ability to understand and use mathematics in later school years.

Early mathematics learning can challenge children to explore ideas about patterns and relationships, order and predictability, and measurement. Supportive instruction occurs in classroom environments that are conversationally rich, encourage children's thinking, and respect and nurture children's explorations. It is important to keep in mind that mathematics appears in all aspects of children's play experiences so that the support you provide children will reinforce and extend children's understandings in an authentic way.

The International Academy of Education (IAE) emphasizes ten principles of effective mathematics teaching that you can apply to your preschool classrooms. The IAE principles are listed in column 1 of the following table; you will also find associated action steps preschool teachers can take to support children's mathematics learning experiences.

Principles and Action Steps for Effective Preschool Math Teaching

<i>Teaching Principles</i>	<i>Action Steps for Preschool Classrooms</i>
Caring classroom communities that are focused on mathematical goals help develop students' mathematical identities and proficiencies.	• Build a community of trust where young children feel safe asking questions and exploring ideas. • Build classroom math experiences around clearly defined goals and aims.
Effective teachers provide students with opportunities to work both independently and collaboratively to make sense of ideas.	• Provide a balance of mathematical learning experiences for the individual student, small groups, and the whole class.
Effective teachers plan mathematics learning experiences that enable students to build on their existing proficiencies, interests, and experiences.	• Connect classroom mathematics experiences to children's existing understandings and knowledge. • Use children's misconceptions and errors as building blocks for developing deeper understandings.
Effective teachers understand that the tasks and examples they select influence how students come to view, develop, use, and make sense of mathematics	• Provide mathematics experiences that encourage original thinking and encourage children to struggle with ideas. • Focus on thinking versus providing the right answer. • Provide children many opportunities to practice what they are learning throughout the day.
Effective teachers support students in creating connections between different ways of solving problems, between mathematical representations and topics, and between mathematics and everyday experiences.	• Provide opportunities for children to use mathematics knowledge in their everyday experiences. • Emphasize the links between different mathematical ideas and learning experiences.

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Teaching Principles	Action Steps for Preschool Classrooms
Effective teachers use a range of assessment practices to make students' thinking visible and to support students' learning.	• Provide opportunities for children to document their own thinking during learning experiences. • Pay close attention to children's progress and take notes to document their thinking. • Take the time to listen to children's responses, ask questions, and provide supportive feedback to all children.
Effective teachers are able to facilitate classroom dialogue that is focused on mathematical argumentation.	• Encourage children to explain their thinking verbally or through drawing and writing.
Effective teachers shape mathematical language by modeling appropriate terms and communicating their meaning in ways students understand.	• Model mathematics language in the classroom.
Effective teachers carefully select tools and representations to provide support for students' thinking.	• Provide children with an appropriate variety of mathematics learning materials.
Effective teachers develop and use sound knowledge as a basis for initiating learning and responding to the mathematical needs of all of their students.	• Build your own understanding of mathematics content and curricular goals to make informed decisions. • Seek out professional development opportunities to build your understandings of content and pedagogy.

Source for column 1: Anthony, Glenda, and Margaret Walshaw. 2009. *Effective Pedagogy in Mathematics*. Educational Practices Series—19. Belgium: International Academy of Education. http://www.ibe.unesco.org/fileadmin/user_upload/Publications/Educational_Practices/EdPractices_19.pdf

Guided Inquiry

Inquiry-based learning can play a central role in the development of meaningful learning opportunities as children explore emerging skills in early mathematics. Contrary to traditional notions of the teacher's role as a teller of information, teachers in inquiry classrooms perform the roles of guide, facilitator, and provocateur by asking questions and designing meaningful lessons built on student interests. A teacher's ability to listen to her students builds the foundation for guided inquiry in the classroom.

When using an inquiry-driven approach to mathematics learning, a teacher works closely with children to support observation and listening skills, encourage individual expression, and promote problem exploration. This approach requires planning and intensive teaching on the part of the teacher as well as attentiveness and active engagement on the part of the children. As teachers work to pose questions to prompt interest, children will work to delve further into their understandings through cycles of questioning, exploration, and documentation. In inquiry-driven classrooms, teacher questioning patterns associated with traditional models of teaching are set aside in favor of open-ended, exploratory questioning. Instead of asking who can tell what the shape is named, you might ask



the children to describe the shape to you or ask them how they know it is a certain shape. You then could promote deeper thinking by asking the children what they would like to know about the shape. As you move away from the “guess what’s in my head” questioning patterns, you can

model for the children a higher level of questioning that encourages them to think about what they know and don't know, when they need to find more information, and whether new information supports or extends their previous understandings.

The concept of *possibility thinking* encourages teachers to consider how questions, play, supportive classrooms, imagination, innovation, risk taking, and self-determination—core elements identified by researchers Teresa Cremin, Pamela Burnard, and Anna Craft—affect the processes of thinking and learning. Possibility thinking involves a dynamic interplay between children and teachers in the following ways:

- **Questioning**—Teachers acknowledge and celebrate children's questions. Teachers pose questions that encourage inquiry.
- **Play**—Children have opportunities for extended play periods.
- **Immersion**—Children are immersed in a benign environment free from criticism and mockery.
- **Innovation**—Teachers closely observe innovations in children's thinking in order to prompt and encourage.
- **Imagination**—Children have ample opportunities to meld imagination and classroom learning.
- **Self-determination and risk taking**—Teachers encourage deep involvement and risk taking, and the children support these behaviors.

The lessons you encounter in this book encourage you to carefully consider your interactions with young children as well as the classroom environment you create together. The interplay among children, teachers, and the classroom environment are all central to the process of learning. The following table suggests action steps that can help you in creating positive and creative math learning experiences for preschoolers.

Promoting Guided Inquiry and Creative Math Learning

<i>Classroom Components</i>	<i>Teacher Actions</i>
Physical environment	• Thoughtfully include a variety of manipulatives, blocks, natural materials, and digital media for free exploration.
Role of the teacher	• Develop a supportive environment for playful learning, experimentation, and risk taking. • Closely observe children’s play and exploration, using formative assessments. • Ask thoughtful questions and provide provocations to expand and clarify children’s thinking.
Relationships among peers	• Provide opportunities for collaborative experiences. • Demonstrate respect for children’s work. • Promote opportunities for play and exploration.
Structure of mathematics lessons and experiences	• Provide opportunities for individual and group experiences. • Maintain flexible scheduling for lesson lengths based on children’s responses and interests. • Provide for repeated mathematics experiences. • Promote opportunities for children to make their thinking visible (using concrete manipulatives, math journals, digital photography, and so on). • Extend familiar lessons and concepts to build proficiency and flexibility of student understanding.

Organization of the Book

Setting up the classroom environment to encourage math explorations is important. Chapter 1 provides guidance on organizing the math center and the classroom. Each of the following chapters is based on a particular mathematics content area: numbers and number sense, computation, geometry and spatial sense, measurement, data collection and statistics, and patterns and relationships. In each chapter, you will see a listing of the National Council of Teachers of Mathematics (NCTM) content learning standards. In addition to the particular content information, NCTM recommends that teachers provide opportunities for children to meet process standards in the areas of problem solving, reasoning and proof, communication, connections, and representation. These process standards align with the inquiry-driven, creative mathematics experiences recommended throughout this book. Alongside the NCTM process standards and expectations, you will see more-specific approaches for supporting preschool math learning in those areas.

Supporting NCTM Process Standards

<i>NCTM Process Standard</i>	<i>Expectations for Students</i>	<i>Supportive Approaches in the Early Childhood Classroom</i>
Problem solving	• Build new mathematical knowledge through problem solving. • Solve problems that arise in mathematics and in other contexts. • Apply and adapt a variety of appropriate strategies to solve problems. • Monitor and reflect on the process of mathematical problem solving.	Provide opportunities for children to ask questions, work out planning and trials, work in pairs and small groups, and plan experiments.

<i>NCTM Process Standard</i>	<i>Expectations for Students</i>	<i>Supportive Approaches in the Early Childhood Classroom</i>
Reasoning and proof	• Recognize reasoning and proof as fundamental aspects of mathematics. • Make and investigate mathematical conjectures. • Develop and evaluate mathematical arguments and proofs. • Select and use various types of reasoning and methods of proof.	Provide opportunities for children to document their thinking through drawing and writing and to share their thoughts with large and small groups.
Communication	• Organize and consolidate their mathematical thinking through communication. • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others. • Analyze and evaluate the mathematical thinking and strategies of others. • Use the language of mathematics to express mathematical ideas precisely.	Provide opportunities for children to share their problem-solving experiences, and encourage and support children's use of mathematics language.
Connections	• Recognize and use connections among mathematical ideas. • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. • Recognize and apply mathematics in contexts outside of mathematics.	Provide opportunities for children to use math in everyday experiences and in other content areas. Revisit math lessons multiple times to encourage mastery and promote confidence.

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NCTM Process Standard	Expectations for Students	Supportive Approaches in the Early Childhood Classroom
Representation	• Create and use representations to organize, record, and communicate mathematical ideas. • Select, apply, and translate among mathematical representations to solve problems. • Use representations to model and interpret physical, social, and mathematical phenomena.	Provide opportunities for children to represent their thinking through drawing, writing, and verbal experiences.

Source: National Council of Teachers of Mathematics. 2000. *Principles and Standards for School Mathematics*. Reston,VA: NCTM. Adapted with permission.

This book shares practical strategies designed to help you encourage preschoolers’ learning of mathematics processes and content. At the beginning of chapters 2 through 7, you will find information that will familiarize you with the particular chapter topic. You will also find classroom vignettes to help bring the content and teaching strategies to life. Woven into each of those chapters are preschool math lessons based on best practices for guided inquiry and creative thinking. You will also find guidance on content-specific math centers, whole-group experiences, and recommended children’s books related to each chapter’s content.

1

Developing a Creative Math Environment

Early childhood educators have essential roles in the development of children's creative-thinking skills because they can either create supportive classroom environments or classrooms in which children's creative skills are stifled. Teachers can incorporate creative learning experiences by designing lessons that include opportunities for critical thinking and reflection while also maintaining a focus on student interest. In a classroom environment, creativity should be viewed as a learning process that encourages social interaction, promotes individual ownership of ideas, and requires children to document and report on their thinking and experiences. When young children have opportunities to personally engage with challenging, reflective learning experiences, they can build their critical- and creative-thinking skills.

The lesson ideas and classroom vignettes shared throughout this book incorporate opportunities to help children understand the concepts of *numeracy*, *classification*, *comparing*, *sequencing*, *shape*, *structure*, *location*, *motion*, and *transformation*. The lessons include critical elements of inquiry and creative thinking—open-ended tasks, activities involving social interaction, and opportunities for children's reflection and elaboration. In other words, the lessons in this book will encourage you to plan and implement math experiences that teach children ways to think rather than narrowly focus on teaching children what to think. Open-ended tasks provide young learners with opportunities to experiment with new ideas and engage in inquiry. Because open-ended tasks promote idea experimentation, they encourage children to focus on the processes of learning rather than the need to arrive at a single correct answer. Gaining experience with idea experimentation can help children accept ambiguity,

be willing to make mistakes, and gain confidence in their problem-solving abilities. Likewise, providing opportunities for small-group work and social interaction is a crucial component of creative thinking. Working in pairs or small groups will help to promote brainstorming and allow children to learn from and with each other. Such tasks will also support children's experiences with reflection and idea elaboration. These skills are important cognitive tools that allow preschoolers to learn from their own experiences and examine their own learning process. Employing these components of creativity in the classroom will help to create a rich, engaging learning environment for all of the children.



Designing the Math Learning Center

In addition to planning and implementing targeted mathematics learning experiences in early childhood classrooms, it is important to structure the classroom environment to support children's everyday explorations in numeracy, geometry, estimation, problem solving, and measurement. The math center can be a permanent or moveable area in your classroom where math materials and manipulatives are available for children to freely explore. Although the center does not need to be large, it is important for the materials to be engaging and well-organized to support children's independent explorations. Consider the following elements for your math center:

- **Work space**—Ensure that children have enough space to use materials and manipulatives without becoming frustrated. For example, puzzles can be an effective way for children to experiment with size, directionality, and patterning, but children will need to have room to lay out all of the pieces and explore them for an extended period of time. The size of your mathematics center will also determine whether children will work individually, in pairs, or in small groups.
- **Storage and display of mathematics materials**—If you have enough room, the math center location can also be used to store mathematics materials permanently for children and teachers to easily access. If space is limited, materials can be easily rotated in and out to support the children's current interests and curricular experiences. If you have wall space near the math center, use it to display the charts, graphs, and children's work generated during classroom mathematics lessons. You can encourage children to use these displays to support and extend their explorations.
- **Books and math journals**—Place relevant children's mathematics books in the center in addition to the children's math journals.



An intentional and well-designed mathematics center will provide children opportunities to extend and continually explore the concepts introduced during math lessons at their own pace and interest level. Revisiting ideas and concepts during play is important to the process of knowledge building. Design your center so that children can do the following:

- Explore and learn based on their interests
- Engage in discovery and construction of meaning
- Extend activities from the lessons
- Explore concepts from the lessons or related concepts in depth
- Connect math to daily experiences

Ideas for Managing the Center

You can optimize children's individual and small-group work in the math center in these ways:

- Involve the children in the setup and maintenance of the center. Carefully organize the materials so that the children can help keep them organized after use.
- Carefully consider the number of children that the center can support at a given time.



- Ensure that children have enough work time while in the center. Exploration takes time. Provide them with repeated opportunities to return to the center.
- Share and discuss your expectations that children respect and use materials carefully.
- Ensure that children have opportunities to share and discuss their center-based explorations and discoveries. Math journals are an effective tool for children to document their work.

Materials and Manipulatives for Math Explorations

Consider stocking the following types of items in the center:

- **Numbers and Number Sense**—counters of various sizes and colors; sorting trays; dice; abacuses; number boards; glass cabochons of various sizes, colors, and shapes; and natural materials such as leaves, pinecones, seashells, and rocks of various sizes and colors
- **Computation**—a cash register, realistic coins and paper money, pads of paper and pencils for recording numbers, and counters of various sizes and colors
- **Geometry and Spatial Sense**—puzzles, shape blocks of various sizes and colors, maps, and materials for mapmaking
- **Measurement**—measuring tape, rulers, balances, clocks, calendars, and materials for nonstandard measurement such as string
- **Patterns and Relationships**—sorting sets and counters of various shapes and colors, and other sorting items such as rocks, seashells, mosaic tiles, and buttons
- **Data Collection and Analysis**—various collections of items and containers for sorting

In addition to these materials, digital technologies can be a valuable part of early mathematics learning. These may include tablet and computer-based apps and games, interactive whiteboards, and digital cameras.

Providing access to a variety of materials beyond traditional resources can broaden and extend children's explorations. Consider the theory of *loose parts* proposed by architect Simon Nicholson in the 1970s. Loose parts are materials without a predetermined purpose that can be moved, carried, combined, redesigned, and taken apart and put back together in multiple ways. Loose parts can be used alone or combined with other materials and can be both man-made and natural. In your preschool classroom, loose parts that can be used in geometry and spatial explorations might include blocks, sand, playdough, recycled paper tubes, paper scraps, ribbons, lids, wood scraps, wire, cardboard, large buttons, glass cabochons, string, sticks, large beads, and straws. Take care to include only items that are appropriate for your students and that will not present choking hazards. Including loose parts as part of early geometric explorations promotes independent investigations and allows children opportunities to manipulate objects. Expanding children's access to nontraditional materials can provide a wider range of learning opportunities in the classroom. Many of the examples from early childhood classrooms shared in the following chapters involve a combination of traditional mathematics learning materials, digital technologies, and loose parts.